



SYDNEY BOYS HIGH
MOORE PARK, SURRY HILLS

2006
YEAR 10 HALF-YEARLY
EXAMINATION

Advanced Mathematics

Directions to Candidates:

- Answer all questions in the spaces provided in this question booklet.
- Full marks may not be awarded for careless or badly arranged work.
- Use black or blue pen for written answers, but pencil for diagrams and graphs.

- If additional working space is needed, use the spare pages at the end of the booklet. Show clearly which question you are continuing.
- Board-approved calculators may be used.

Time allowed: 90 minutes

Examiner: Mr C. Kourtesis

Name: _____

Your Mathematics Class (Tick the box)	
10Ma A Mr Fuller	<input type="checkbox"/>
10Ma B Ms Ward	<input type="checkbox"/>
10Ma C Mr Boros	<input type="checkbox"/>
10Ma D Ms Evans	<input type="checkbox"/>
10Ma E Mr McQuillan	<input type="checkbox"/>
10Ma F Mr Gainford	<input type="checkbox"/>

Markers' Use Only	
Question 1	/20
Question 2	/20
Question 3	/15
Question 4	/15
Question 5	/15
Question 6	/15
Total	/100

Question 1 (20 marks)

Answers

Marks

- (a) Express $\frac{1}{9}$ as a decimal. 1
- (b) Simplify $4ab \times 2a$. 1
- (c) Find $8^{2.3}$ correct to 2 decimal places. 2
- (d) Simplify $16m - 2 - 9m + 5$. 1
- (e) Express 543 000 000 in standard notation. 1
- (f) If $a = 3$ and $b = 4$, evaluate ab^2 . 1
- (g) Factorise $6xy + 2x$. 1
- (h) Solve the following:
- (i) $\frac{5k}{4} = 9$ 1
- (ii) $4x = 1 - 3x$ 2

(i) Expand and simplify $(3x + 4)^2$. 2

(j) Simplify
(i) $2\sqrt{7} + 3\sqrt{5} + 4\sqrt{7}$ 1

(ii) $\sqrt{8} + \sqrt{12}$ 2

(k) Express 80 km/h in m/s. 1

(l) Find the area of a circle of radius 10 cm.
(Answer in terms of π .) 1

(m) Simplify $(3\sqrt{5})^2$. 1

(n) A fair, six-sided die is cast. What is the probability of obtaining a 2 or a 3? 1

Question 2 (20 marks)**Answers****Marks**

(a) Solve the equation $(x - 3)(2x + 1) = 0$.

2

(b) Simplify $\frac{m - 4}{5} + \frac{3m}{7}$.

2

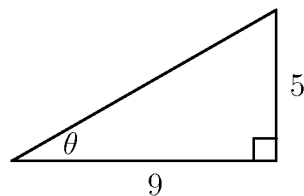
(c) Find the midpoint of the interval joining $A(10, 4)$ and $B(-6, -1)$.

2

(d) Find the gradient of the line $2x + 3y = 5$.

2

(e) Find the size of θ correct to the nearest minute.

2

(f) Solve the inequality $4 - 3x \geq -5$. 2

(g) Find the simple interest earned if \$7430 is invested for 5 years 5 months at $9\frac{1}{4}\%$ p.a. 2

(h) If $\frac{3^{-2}}{4^{-3}} = \frac{a}{b}$, find the values of a and b where a and b are whole numbers. 2

(i) Express $\frac{1}{\sqrt{3} + 5}$ with a rational denominator. 2

(j) A cylinder has a base radius of 7 m and a height of 20 m.
Find its:

(i) volume (in terms of π), 1

(ii) capacity in litres (nearest litre). 1

Question 3 (15 marks)**Answers****Marks**

- (a) The point $(-5, k)$ lies on the line $6x + 3y = 2$.
Find the value of k .

2

- (b) Solve the following equations:

(i) $x^2 = 7x$

1

(ii) $x^2 - 2x = 8$

2

(iii) $(5k - 1)^2 = 9$

2

(iv) $2x^2 + 7x - 1 = 0$

2

- (c) Find the amount of compound interest earned if \$8400 is invested for 9 years at 12.5% p.a., compounded monthly.

2

- (d) John borrows \$20 000 which is to be repaid by equal monthly instalments over 7 years. Simple interest is charged at 6% p.a.

(i) How much is repaid altogether?

2

(ii) What is each monthly instalment?

2

Question 4 (15 marks)

(a) If the birth of a boy or girl are equally likely events, find the probability that in 3 separate births the babies are:

(i) all boys,

1

(ii) at least two girls.

1

(b) A man buys 4 tickets in a raffle in which there are two prizes and 100 tickets sold. Find the probability that he wins

(i) both prizes,

1

(ii) only the second prize,

2

(iii) only one prize,

2

(c) If each edge of a cube is increased by 150%, what is the percentage increase in the surface area?

2

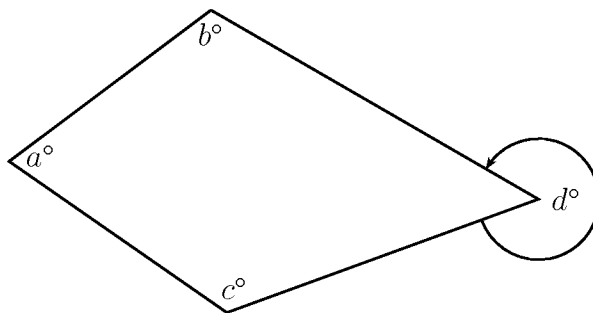
- (d) Solve the quadratic equation $2x^2 - 4x - 1 = 0$ by using the method of “completing the square.”

2

- (e) A person’s wage was increased by $a\%$ to \$250 per week. What was the wage, in dollars, just prior to the increase?

2

- (f)



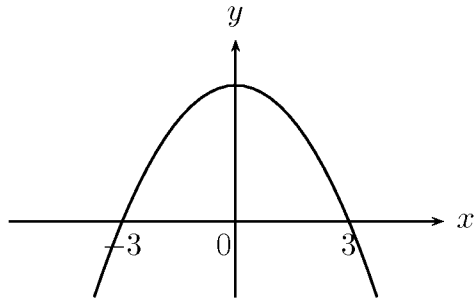
2

Find an equation connecting a , b , c , and d .

Question 5 (15 marks)

- (a) A sketch of $y = -x^2 + w$ is given below.

1



Find the value of w .

- (b) Write down the equation of the circle with radius 10 units and centre $(0, 0)$.

1

- (c) Express $V = \sqrt{2GM \left(\frac{1}{r} - \frac{1}{R} \right)}$ with R as the subject.

3

(d) Factorise fully $ab(p^2 + q^2) + pq(a^2 + b^2)$.

3

(e) Given the equation of the parabola $y = x^2 + x - 2$,
find the

(i) x and y intercepts,

3

(ii) coördinates of the vertex,

1

(iii) coördinates of the points of intersection of
the parabola with the line $y = x - 1$.

3

Question 6 (15 marks)

- (a) Three students, A , B , and C , try out for a football team. The chances that A , B , and C make the team are 60% , $66\frac{2}{3}\%$, 75% respectively.

Calculate the probability that

- (i) all 3 make the team,

1

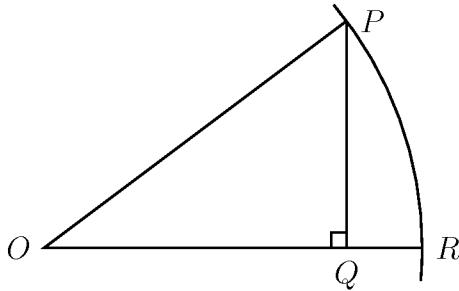
- (ii) all miss out on making the team,

1

- (iii) only A and B make the team.

2

- (b)



3

P and R are points on a circle with centre O .
 $PQ = 50$ cm and $QR = 10$ cm. Find the radius of the circle.

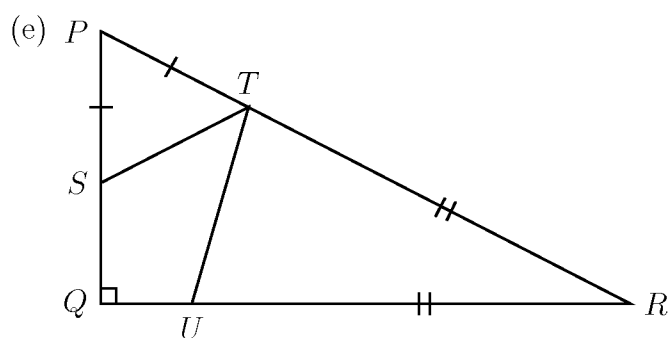
(c) Solve the equation

3

$$\frac{1}{u-1} - \frac{1}{u-2} = 4$$

(d) A 5% salt water solution is one which is made up of 5% salt and 95% water. How much water has to be added to 1 litre of 5% salt-water solution in order to produce a 4% salt-water solution?

2



3

Triangle PQR is right angled at Q and triangles PST and RTU are isosceles as shown. If $\angle STU$ measures x° , then the value of x is

End of Paper

Extra working page

Extra working page

Extra working page

Question 1 (20 marks)

Answers

Marks

(a) Express $\frac{1}{9}$ as a decimal.

$$0.\dot{1}$$

1

(b) Simplify $4ab \times 2a$.

$$8a^2b$$

1

(c) Find $8^{2.3}$ correct to 2 decimal places.

$$119.43$$

2

(d) Simplify $16m - 2 - 9m + 5$.

$$7m + 3$$

1

(e) Express 543 000 000 in standard notation.

$$5.43 \times 10^8$$

1

(f) If $a = 3$ and $b = 4$, evaluate ab^2 .

$$48$$

1

(g) Factorise $6xy + 2x$.

$$2x(3y+1)$$

1

(h) Solve the following:

(i) $\frac{5k}{4} = 9$

$$5k = 36$$

$$k = \frac{36}{5}$$

1

(ii) $4x = 1 - 3x$

$$7x = 1$$

$$x = \frac{1}{7}$$

2

(i) Expand and simplify $(3x + 4)^2$.

$$9x^2 + 24x + 16$$

2

(j) Simplify

(i) $2\sqrt{7} + 3\sqrt{5} + 4\sqrt{7}$

$$6\sqrt{7} + 3\sqrt{5}$$

1

(ii) $\sqrt{8} + \sqrt{12}$

$$2\sqrt{2} + 2\sqrt{3} \\ = 2(\sqrt{2} + \sqrt{3})$$

2

(k) Express 80 km/h in m/s.

$$80 \frac{\text{km}}{\text{hr}} = \frac{80 \times 1000\text{m}}{3600\text{s}}$$

$$= 22.2 \text{ m/s}$$

1

(l) Find the area of a circle of radius 10 cm.
(Answer in terms of π .)

$$A = \pi r^2 \\ = \pi \times 10^2$$

$$= 100\pi \text{ cm}^2$$

1

(m) Simplify $(3\sqrt{5})^2$.

$$45$$

1

(n) A fair, six-sided die is cast. What is the probability of obtaining a 2 or a 3?

$$\frac{1}{3}$$

1

Question 2 (20 marks)

Answers

Marks

- (a) Solve the equation $(x - 3)(2x + 1) = 0$.

2

$$x = 3 \quad \text{or} \quad x = -\frac{1}{2}$$

- (b) Simplify $\frac{m - 4}{5} + \frac{3m}{7}$.

2

$$\frac{7m - 28}{35} + \frac{15m}{35} = \frac{22m - 28}{35}$$

- (c) Find the midpoint of the interval joining $A(10, 4)$ and $B(-6, -1)$.

2

$$\text{mp} \left(2, \frac{3}{2} \right) \quad \text{or} \quad \left(2, 1\frac{1}{2} \right)$$

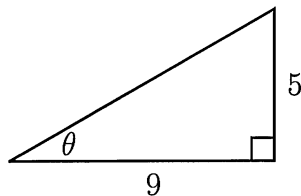
- (d) Find the gradient of the line $2x + 3y = 5$.

2

$$-\frac{2}{3}$$

- (e) Find the size of θ correct to the nearest minute.

2



$$\tan \theta = \frac{5}{9}$$

$$\theta = 29^{\circ} 3'$$

- (f) Solve the inequality $4 - 3x \geq -5$.

$$\begin{aligned} -3x &\geq -9 \\ x &\leq 3. \end{aligned}$$

2

- (g) Find the simple interest earned if \$7430 is invested for 5 years 5 months at $9\frac{1}{4}\%$ p.a.

$$\begin{aligned} I &= PRT \\ &= 7430 \times \frac{9.25}{100} \times (65). \\ &= \$3722.74 \end{aligned}$$

2

- (h) If $\frac{3^{-2}}{4^{-3}} = \frac{a}{b}$, find the values of a and b where a and b are whole numbers.

$$\frac{1}{9} \div \frac{1}{64} = \frac{64}{9}$$

$$a = 64$$

$$b = 9.$$

2

- (i) Express $\frac{1}{\sqrt{3}+5}$ with a rational denominator.

$$\begin{aligned} \frac{1}{\sqrt{3}+5} \times \frac{\sqrt{3}-5}{\sqrt{3}-5} &= \frac{\sqrt{3}-5}{3-25} \\ &= -\frac{\sqrt{3}-5}{22}. \end{aligned}$$

2

- (j) A cylinder has a base radius of 7 m and a height of 20 m. Find its:

- (i) volume (in terms of π),

$$V = \pi 7^2 \times 20$$

$$= \pi 49 \times 20$$

$$= 980\pi \text{ m}^3$$

- (ii) capacity in litres (nearest litre).

$$980\pi \times 1000 = 3078761 \text{ L.}$$

1

1

Year 10 Solutions

Question 3 (15 marks)

Answers

Marks

- (a) The point $(-5, k)$ lies on the line $6x + 3y = 2$.
Find the value of k .

2

$$(6 \times -5) + 3k = 2 \quad - \textcircled{1}$$

$$-30 + 3k = 2$$

$$3k = 32$$

$$k = \frac{32}{3} = 10\frac{2}{3} \quad - \textcircled{1}$$

- (b) Solve the following equations:

(i) $x^2 = 7x$

$$x^2 - 7x = 0 \Rightarrow x(x-7) = 0$$

1

$$x = 7.$$

$\textcircled{1}$

$$x = 0$$

(ii) $x^2 - 2x = 8$

2

$$x^2 - 2x - 8 = 0$$

$$(x-4)(x+2) = 0 \quad - \textcircled{1}$$

$$x-4=0, \quad x+2=0$$

$$x=4, \quad x=-2. \quad - \textcircled{1}$$

(iii) $(5k-1)^2 = 9$

2

$$5k-1 = \pm 3 \quad - \textcircled{1}$$

$$5k = 4 \text{ or } 2$$

$$k = \frac{4}{5}, \frac{2}{5} \quad - \textcircled{1}$$

(iv) $2x^2 + 7x - 1 = 0$

2

$$\textcircled{1} \quad x = \frac{-7 \pm \sqrt{7^2 - 4 \times 2 \times -1}}{2 \times 2} \quad \left| \quad x = \frac{-7 \pm \sqrt{57}}{4} \quad - \textcircled{1}$$

$$= \frac{-7 \pm \sqrt{49+8}}{4}$$

- (c) Find the amount of compound interest earned if \$8400 is invested for 9 years at 12.5% p.a., compounded monthly.

2

$$\begin{aligned}
 A &= P(1+r)^n \\
 &= 8400 \left(1 + \frac{12.5\%}{12}\right)^{9 \times 12} \quad \leftarrow -\frac{1}{2} \text{ for an error} \\
 &= 8400 \left(1 + \frac{12.5\%}{12}\right)^{108} \\
 &= 25723.70 : (2dp) \quad \text{--- } \textcircled{1}
 \end{aligned}$$

$$\begin{aligned}
 \text{Interest Earned} &= 25723.70 - 8400 \quad \text{--- } \textcircled{1} \\
 &= \$17323.70
 \end{aligned}$$

- (d) John borrows \$20 000 which is to be repaid by equal monthly instalments over 7 years. Simple interest is charged at 6% p.a.

- (i) How much is repaid altogether?

2

$$\begin{aligned}
 I &= PRT \\
 &= 20000 \times \frac{6\%}{100} \times 7 \\
 &= 8400 \quad \text{--- } \textcircled{1}
 \end{aligned}$$

$$\begin{aligned}
 \text{Amount Repaid} &= \$20000 + \$8400 \\
 &= \$28400 \quad \text{--- } \textcircled{1}
 \end{aligned}$$

- (ii) What is each monthly instalment?

2

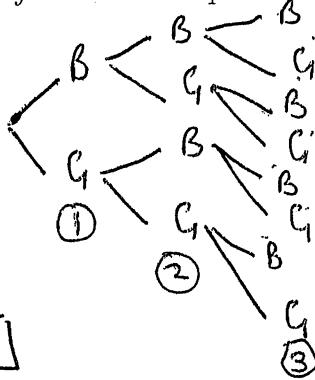
$$\begin{aligned}
 M.I. &= \frac{\$28400}{7 \times 12} \quad \text{--- } \textcircled{1} \\
 &= \$338.10 \quad (2dp). \quad \text{--- } \textcircled{2}
 \end{aligned}$$

Question 4 (15 marks)

(a) If the birth of a boy or girl are equally likely events, find the probability that in 3 separate births the babies are:

(i) all boys,

$$\frac{1}{8} \quad \boxed{1}$$



1

(ii) at least two girls.

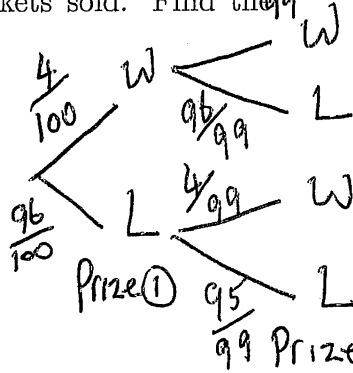
$$\frac{4}{8} = \frac{1}{2} \quad \boxed{1}$$

1

(b) A man buys 4 tickets in a raffle in which there are two prizes and 100 tickets sold. Find the probability that he wins

(i) both prizes,

$$\frac{4}{100} \times \frac{3}{99} = \frac{1}{825} \quad \boxed{1}$$



1

(ii) only the second prize,

$$\frac{96}{100} \times \frac{4}{99} = \frac{32}{825} \quad \boxed{2}$$

2

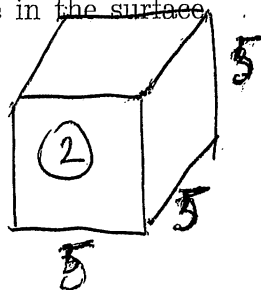
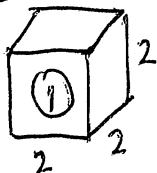
(iii) only one prize,

$$WL + LW = \frac{4}{100} \times \frac{96}{99} + \frac{96}{100} \times \frac{4}{99} = \frac{64}{825} \quad \boxed{2}$$

2

(c) If each edge of a cube is increased by 150%, what is the percentage increase in the surface area?

Take example



$$\begin{aligned} SA \textcircled{1} &= 6 \times 2^2 = 24 \\ SA \textcircled{2} &= 6 \times 5^2 = 150 \\ \text{difference} &= 150 - 24 = 126 \\ \frac{126}{24} \times \frac{100}{1} \% &= 525\% \quad \boxed{2} \end{aligned}$$

2

- (d) Solve the quadratic equation $2x^2 - 4x - 1 = 0$ by using the method of "completing the square." 2

$$\left(-\frac{2}{2}\right)^2$$

$$\begin{aligned} 2x^2 - 4x &= 1 \\ x^2 - 2x &= \frac{1}{2} \\ x^2 - 2x + 1 &= \frac{1}{2} + 1 \\ (x-1)^2 &= \frac{3}{2} \\ x-1 &= \pm \sqrt{\frac{3}{2}} \end{aligned}$$

$$\begin{aligned} x &= 1 \pm \sqrt{\frac{3}{2}} \\ \text{or } &1 \pm \frac{\sqrt{6}}{2} \end{aligned}$$

2

- (e) A person's wage was increased by $a\%$ to \$250 per week. What was the wage, in dollars, just prior to the increase? 2

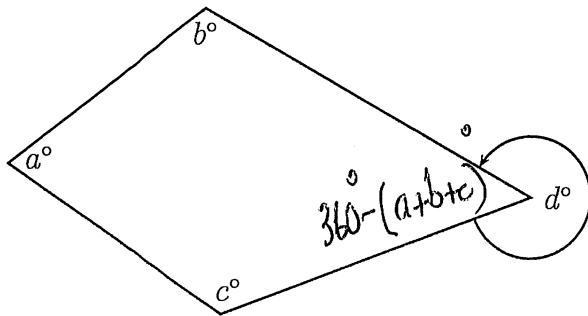
let old wage be \$x

So $\$x + \$x \times a\% = \$250$

$\$x(1 + a\%) = \250

$\$x = \frac{\$250}{1 + a\%}$ 2

- (f) 2



Find an equation connecting a , b , c , and d .

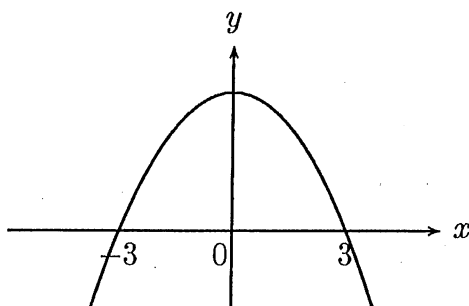
So $d + 360 - (a+b+c) = 360$

$d = a+b+c$ 2

Question 5 (15 marks)

(a) A sketch of $y = -x^2 + w$ is given below.

1



Find the value of w .

$$y = w - x^2 = (3-x)(3+x)$$

$$y = 9 - x^2$$

$$w = 9$$

(b) Write down the equation of the circle with radius 10 units and centre $(0, 0)$.

1

$$x^2 + y^2 = 100$$

(c) Express $V = \sqrt{2GM \left(\frac{1}{r} - \frac{1}{R} \right)}$ with R as the subject.

3

$$V = \sqrt{2GM \left(\frac{1}{r} - \frac{1}{R} \right)}$$

$$V^2 = 2GM \left(\frac{1}{r} - \frac{1}{R} \right)$$

$$\frac{V^2}{2GM} = \left(\frac{1}{r} - \frac{1}{R} \right)$$

$$\frac{V^2}{2GM} - \frac{1}{r} = -\frac{1}{R}$$

$$\frac{Vr^2 - 2GM}{2GMr} = -\frac{1}{R}$$

$$\frac{1}{R} = \frac{2GM - Vr^2}{2GMr}$$

$$R = \frac{2GMr}{2GM - Vr^2}$$

(d) Factorise fully $ab(p^2 + q^2) + pq(a^2 + b^2)$.

$$\begin{aligned} & abp^2 + abq^2 + a^2pq + b^2pq \\ &= ap(bp + aq) + bq(aq + bp) \\ &= (bp + aq)(ap + bq) \end{aligned}$$

(e) Given the equation of the parabola $y = x^2 + x - 2$, find the

(i) x and y intercepts,

when $y=0$ $x^2 + x - 2 = 0$
 $(x+2)(x-1) = 0$
∴ x intercepts are $-2, +1$

when $x=0$ $y = 0 + 0 - 2$
∴ y intercept is -2 .

(ii) coordinates of the vertex,

$$x = -\frac{b}{2a} \quad \therefore x = -\frac{1}{2}$$
$$y = -2\frac{1}{4}$$

Vertex is $(-\frac{1}{2}, -2\frac{1}{4})$.

(iii) coordinates of the points of intersection of the parabola with the line $y = x - 1$.

$$x - 1 = x^2 + x - 2$$

$$0 = x^2 - 1$$

$$x = \pm 1$$

when $x = +1$ $y = 0$ } points of intersection
when $x = -1$ $y = -2$ } are $(1, 0)$ $(-1, -2)$.

Question 6 (15 marks)

- (a) Three students, A , B , and C , try out for a football team. The chances that A , B , and C make the team are 60%, $66\frac{2}{3}\%$, 75% respectively. Calculate the probability that

$$P(A) = \frac{3}{5}$$

$$P(B) = \frac{2}{3}$$

$$P(C) = \frac{3}{4}$$

- (i) all 3 make the team,

$$\frac{3}{5} \times \frac{2}{3} \times \frac{3}{4} = \frac{3}{10}$$

1

- (ii) all miss out on making the team,

$$\frac{2}{5} \times \frac{1}{3} \times \frac{1}{4} = \frac{1}{30}$$

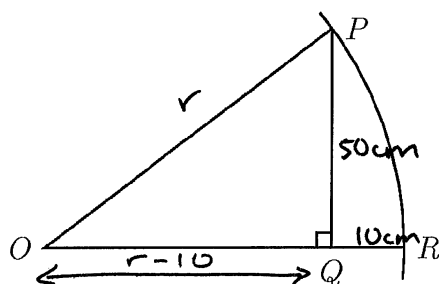
1

- (iii) only A and B make the team.

$$\frac{3}{5} \times \frac{2}{3} \times \frac{1}{4} = \frac{1}{10}$$

2

- (b)



3

P and R are points on a circle with centre O . $PQ = 50$ cm and $QR = 10$ cm. Find the radius of the circle.

$$r^2 = 50^2 + (r-10)^2$$

$$r^2 = 2500 + r^2 - 20r + 100$$

$$20r = 2600$$

$$r = 130 \text{ cm}$$

(c) Solve the equation

3

$$\frac{1}{u-1} - \frac{1}{u-2} = 4$$

$$u-2 - (u-1) = 4(u-1)(u-2)$$

$$\cancel{u}-2-\cancel{u}+1 = 4(u^2-2u-u+2)$$

$$-1 = 4u^2 - 12u + 8$$

$$4u^2 - 12u + 9 = 0$$

$$\frac{(4u-6)^2}{2 \times 2} = 0$$

$$\begin{array}{r} \times 36 \\ + \sqrt{-12} \\ -6, -6 \end{array}$$

$$(2u-3)^2 = 0$$

$$\therefore u = \frac{3}{2}$$

(d) A 5% salt water solution is one which is made up of 5% salt and 95% water. How much water has to be added to 1 litre of 5% salt-water solution in order to produce a 4% salt-water solution?

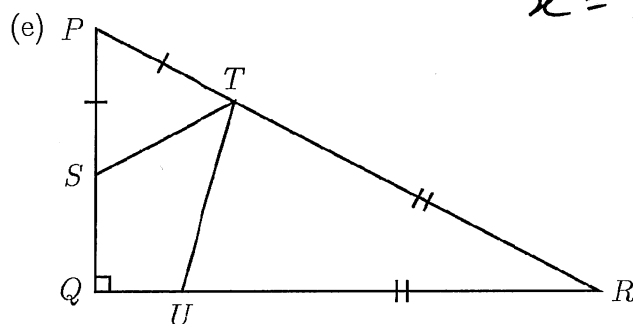
2

$$\frac{50}{1000+x} \times 100 = 4$$

$$5000 = 4000 + 4x$$

$$4x = 1000$$

$$x = 250\text{mL}$$



3

Triangle PQR is right angled at Q and triangles PST and RTU are isosceles as shown. If $\angle STU$ measures x° , then the value of x is

let $\angle PTS = a$
 $\angle PST = a$ (base \angle 's isos. Δ)
 $\angle SPT = 180^\circ - 2a$ (\angle sum of $\Delta SPT = 180^\circ$)
 $\angle PRQ = 2a - 90^\circ$ (\angle sum of $\Delta PRQ = 180^\circ$)
 $\angle RTU = \angle RUT = 135^\circ - a$ (\angle sum of isos. $\Delta URT = 180^\circ$)
 $\angle PTS + \angle STU + \angle UTR = 180^\circ$ (\angle 's on a straight line)
 $a + x + 135^\circ - a = 180^\circ$ End of Paper
 $\therefore x = 45^\circ$