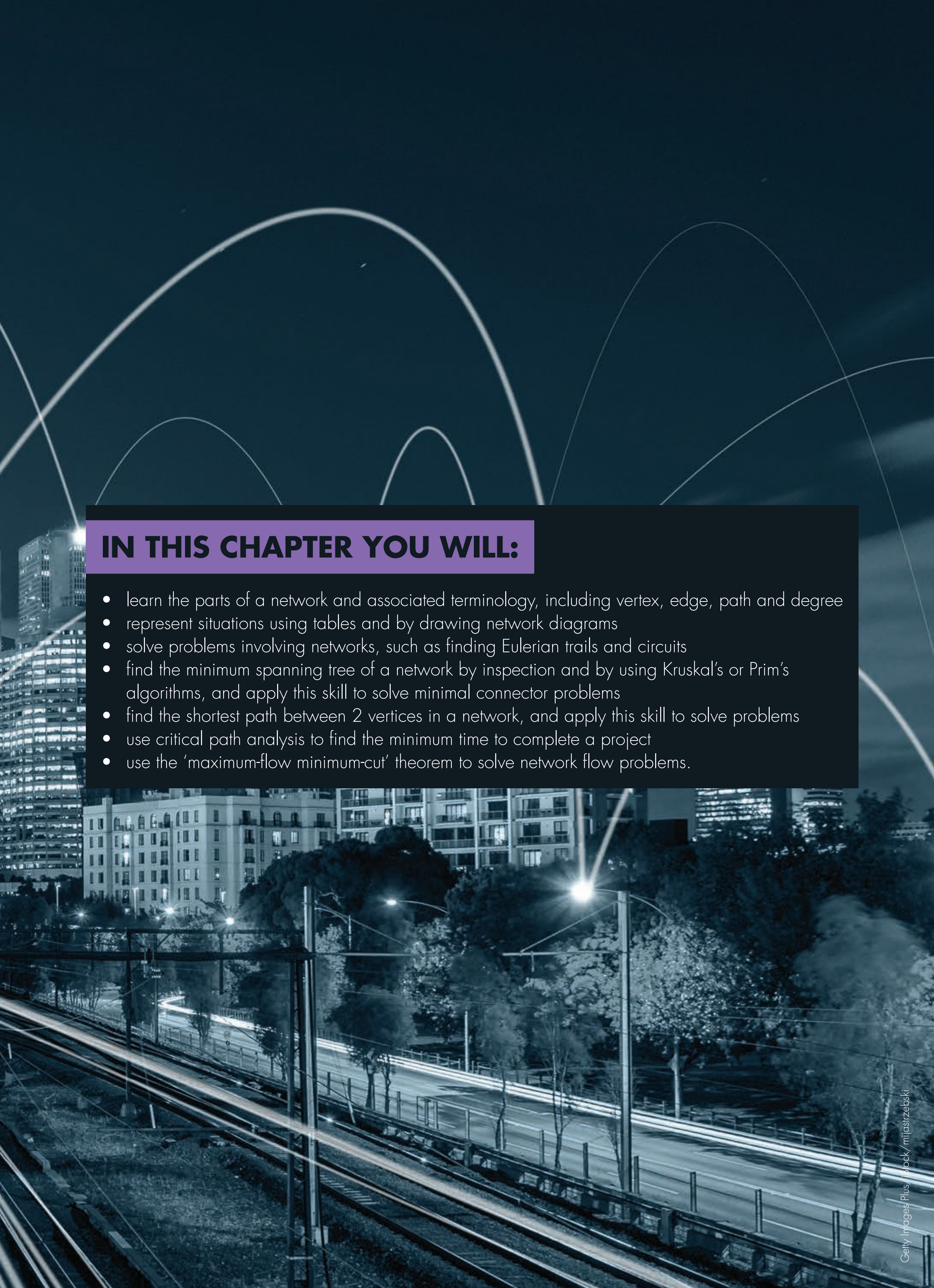
6.

NETWORKS

A **network** is a system or structure of things that are related. Bus, rail and air routes are examples of networks. Mathematicians study networks and use graph theory to try to improve them: faster process times, shorter distances, lower costs. Networks are used to represent computer systems, electrical and plumbing systems for housing, GPS road navigation, garbage bin collection routes and social media connections such as Facebook.

CHAPTER OUTLINE

N2 6.01 Networks
N2 6.02 Eulerian trails and circuits
N2 6.03 Minimum spanning trees
N2 6.04 Shortest path problems
N3 6.05 Activity tables and forward scanning
N3 6.06 Backward scanning and critical path analysis
N3 6.07 Network flow problems
N3 6.08 The 'maximum-flow minimum-cut' theorem



IN THIS CHAPTER YOU WILL:

- learn the parts of a network and associated terminology, including vertex, edge, path and degree
- represent situations using tables and by drawing network diagrams
- solve problems involving networks, such as finding Eulerian trails and circuits
- find the minimum spanning tree of a network by inspection and by using Kruskal's or Prim's algorithms, and apply this skill to solve minimal connector problems
- find the shortest path between 2 vertices in a network, and apply this skill to solve problems
- use critical path analysis to find the minimum time to complete a project
- use the 'maximum-flow minimum-cut' theorem to solve network flow problems.

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TERMINOLOGY

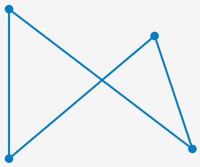
activity table	backward scanning	capacity
circuit	critical activity	critical path
cut	cycle	degree
directed network	dummy activity	earliest starting time (EST)
edge	Eulerian circuit	Eulerian trail
float time	flow capacity	forward scanning
Kruskal's algorithm	latest starting time (LST)	maximum-flow minimum-cut theorem
minimum spanning tree	network	network flow
non-critical activity	path	predecessor
Prim's algorithm	shortest path	sink
source	spanning tree	trail
tree	vertex	vertices
walk	weight	weighted network

WS
Assignment 6

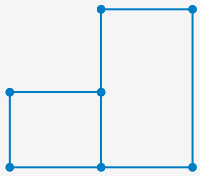
SkillCheck

1 Copy each diagram and using a pencil, trace the diagram along the edges, using each edge only once and without lifting your pencil.

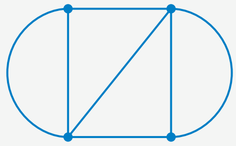
a



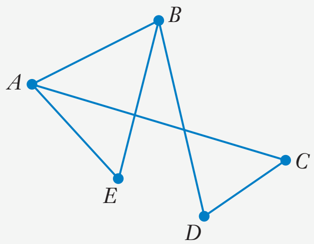
b



c



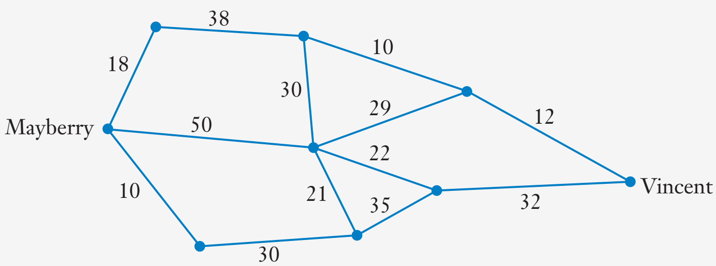
2 This diagram shows the roads linking 5 towns, labelled A, B, C, D and E. Mitch starts from town A, visits each town once and returns to town A.



a Describe a route that Mitch could take.

b List any roads that Mitch does not travel on.

3 This diagram shows the distances, in kilometres, along roads between the towns of Mayberry and Vincent.



Find the shortest distance between Mayberry and Vincent.

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6.01 Networks

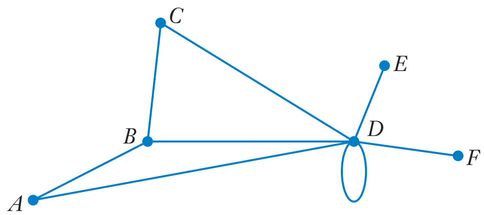
WS Networks terminology

A **network** is a collection of objects connected to each other in some way.

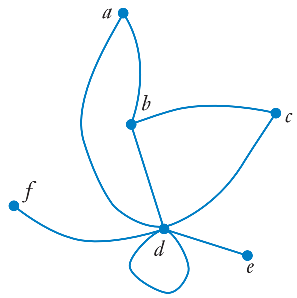
A **graph** is a diagram of a network and contains points called **vertices** (or **nodes**), which are joined by lines called **edges** (or **arcs**). A **loop** is an edge that connects a vertex to itself.

This network has 6 vertices, labelled A to F.

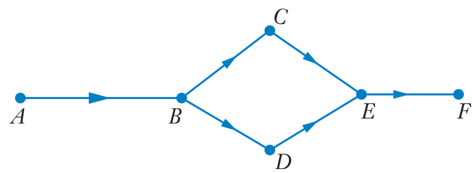
It has 8 edges, including one loop at D.



The same network can also be drawn with curved lines and with the vertices in different positions and labelled by lower-case letters, as long as it shows the same links.

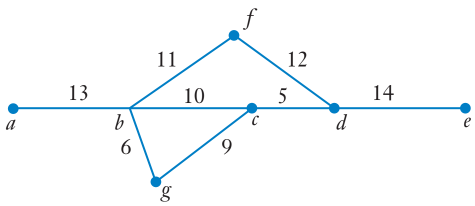


A **directed network** has arrows on its edges that indicate direction.



A **weighted network** has numbers on its edges called **weights**. The numbers could represent distances, times, costs or other quantities.

The **weight** of a network is the sum of the weights of its edges.



EXAMPLE 1

For the weighted network *abcdefg* shown above, find:

a the weight of the network

b the shortest path from a to c

c the longest path from a to c without using an edge more than once.

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Solution

a Weight of network = 13 + 10 + 5 + 14 + 11 + 12 + 6 + 9
= 80

b Shortest path from a to c = 13 + 10
= 23

The path abc

c Longest path = 13 + 11 + 12 + 5
= 41

The path $abfdc$

Degree of a vertex

The **degree** (or **order**) of a vertex is the number of edges connected to it. An **even vertex** has an even number of edges connected to it. An **odd vertex** has an odd number of edges.

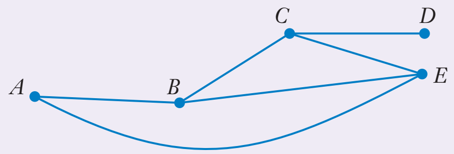
The **sum of the degrees** of all vertices in a network is equal to **twice the number of edges**, because each edge belongs to 2 vertices. This means that the sum of degrees in a network is always even.

EXAMPLE 2

a Find the degree of each vertex in this network.

b How many vertices are even?

c Show that the sum of the degrees is equal to twice the number of edges.



Solution

a

Vertex	A	B	C	D	E
Degree	2	3	3	1	3

b There is one even vertex.

A has degree 2, which is even.

c Number of edges = 6

Sum of degrees of vertices = 12, which is twice the number of edges.

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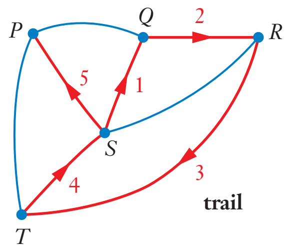
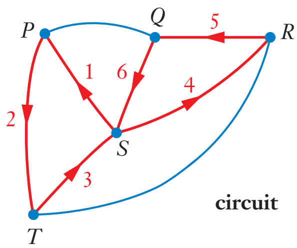
Walks, trails, paths and cycles

A **walk** is any route along the edges of a network, which may include repeated vertices or edges. A walk is **closed** if it starts and ends at the same vertex.

A **trail** is a walk with no repeated edges. A **circuit** is a closed trail.

In the networks below, *SQRTSP* is a **trail** (each edge used once only) and *SPTSRQS* is a **circuit** (returns to *S*), where the numbers show the order. Note however that *S* is used more than once in both cases.

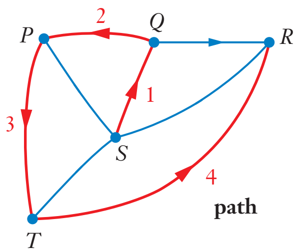


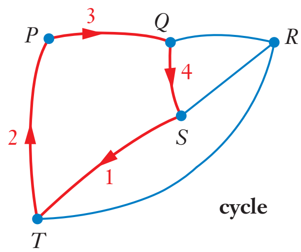


Think of a jogger running along a trail or circuit and not repeating an edge.

A **path** is a walk with no repeated vertices (or edges). A **cycle** is a closed path.

In the networks below, *SQPTR* is a **path** (each vertex used once only) and *STPQS* is a **cycle** (returns to *S*).





Think of a cyclist riding along a path or cycle and not repeating a vertex.

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EXAMPLE 3

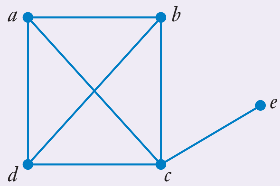
a State whether each walk is a trail or path.

i *abcad* **ii** *abdce* **iii** *adbca*

b Which of the walks in part **a** is a cycle (a closed path)?

c Write down a closed walk that is not a cycle.

d Write down 2 cycles that use the edge *ab*.



Solution

a **i** *abcad* is a trail as no edges are repeated but the vertex *a* is repeated.

ii *abdce* is a path since no vertices (and edges) are repeated.

iii *adbca* is a path since no vertices (and edges) are repeated.

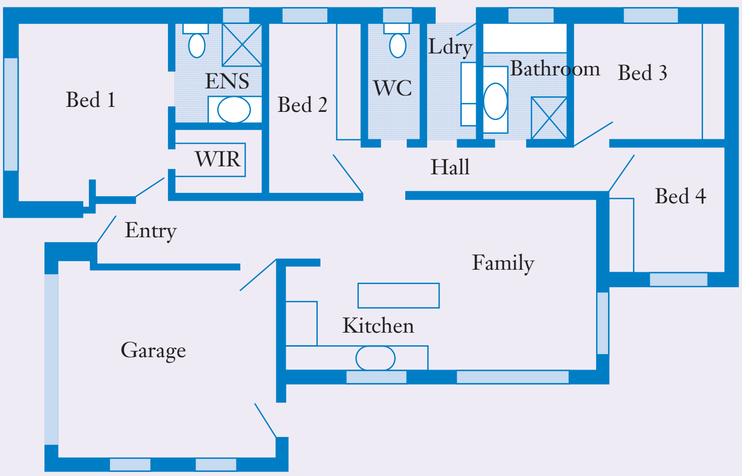
b *adbca* is a cycle.

c *adbcdb* is a path which is not a cycle (the edge *db* is used twice).

d *abdca* and *cabdc* are cycles that use the edge *ab*.

EXAMPLE 4

Model this house plan as a network, showing doors as edges and the rooms (including entry and hall) as vertices.



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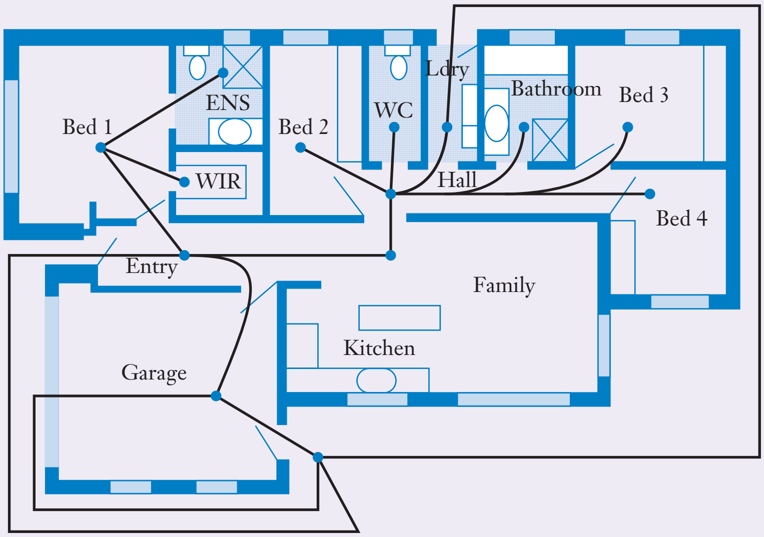
Solution

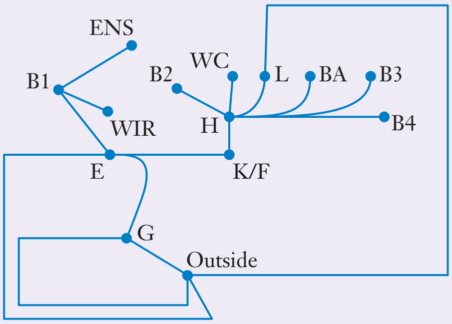
Put a dot over each room (including the entry and hall). Also put one dot outside.

Then connect the dots through the doorways.

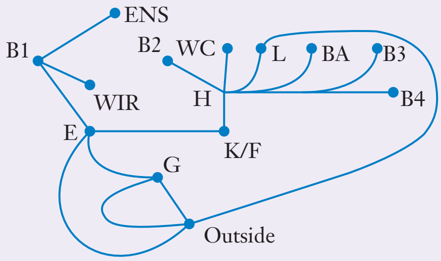


Redraw the network without the plan.

Show connections clearly and label each vertex.



The network could also then be drawn as shown.



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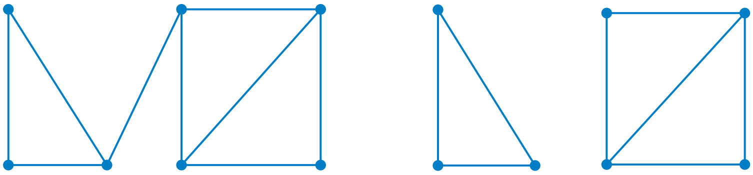
Connected networks

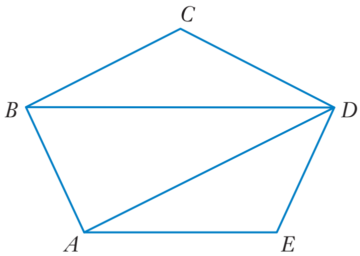
A network is **connected** if there is a path between any 2 vertices on the network.



Connected **Not connected**

Networks and tables

Networks can also be represented using a **matrix** or table of numbers. In the matrix and table below, a 1 means the 2 vertices are joined by an edge and 0 (or '–') means the 2 vertices are not joined by an edge.



diagram/graph

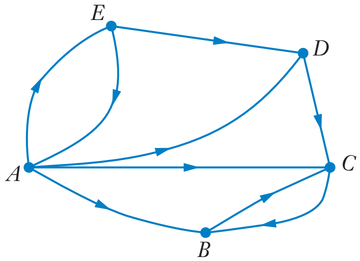
$$\begin{array}{c} \\ \boldsymbol{A} \\ \boldsymbol{B} \\ \boldsymbol{C} \\ \boldsymbol{D} \\ \boldsymbol{E} \end{array} \begin{array}{c} \begin{array}{ccccc} \boldsymbol{A} & \boldsymbol{B} & \boldsymbol{C} & \boldsymbol{D} & \boldsymbol{E} \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

matrix

	A	***B***	***C***	***D***	***E***
A	0	1	0	1	1
B	1	0	1	1	0
C	0	1	0	1	0
D	1	1	1	0	1
E	1	0	0	1	0

table

Directed networks can be shown in a table (or matrix).



		To:				
		A	***B***	***C***	***D***	***E***
From:	***A***	0	1	1	1	1
	B	0	0	1	0	0
	C	0	1	0	0	0
	D	0	0	1	0	0
	E	1	0	0	1	0

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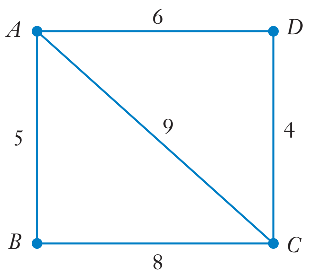
Weighted networks can be shown in a table (or matrix).



network

	A	*B*	*C*	*D*
A	0	5	9	6
B	5	0	8	0
C	9	8	0	4
D	6	0	4	0

table

$$\begin{array}{c} \\ A \\ B \\ C \\ D \end{array} \begin{array}{c} \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{bmatrix} 0 & 5 & 9 & 6 \\ 5 & 0 & 8 & 0 \\ 9 & 8 & 0 & 4 \\ 6 & 0 & 4 & 0 \end{bmatrix} \end{array}$$

matrix

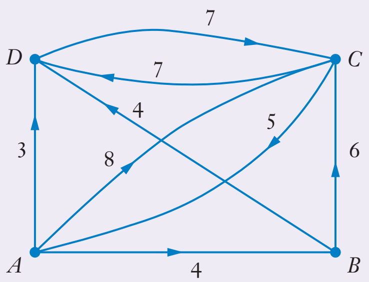
EXAMPLE 5

Draw the directed weighted network represented by this table.

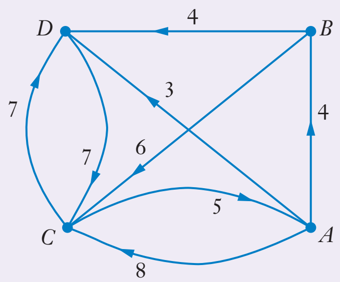
		To:			
		A	***B***	***C***	***D***
From:	***A***	–	4	8	3
	B	–	–	6	4
	C	5	–	–	7
	D	–	–	7	–

Solution

Two possible network diagrams are:



or



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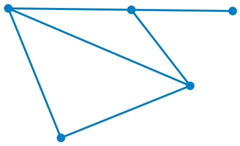
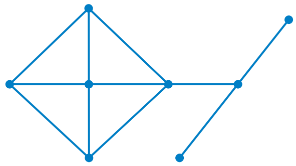
Exercise 6.01 Networks

1 For each network, count the number of: **i** vertices **ii** edges.

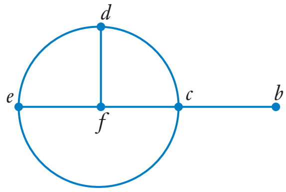
a



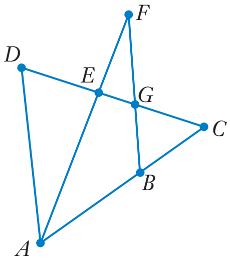
b



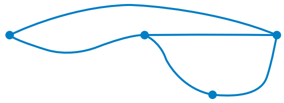
c



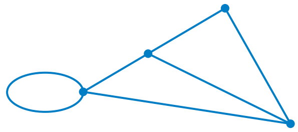
d



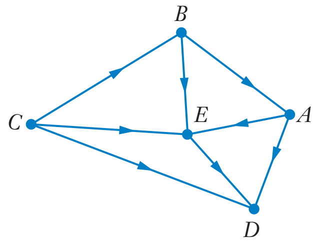
e



f



2 This directed network shows water pipes connecting facilities in a camping ground. Between which 2 facilities can water flow? Select **A**, **B**, **C** or **D**.



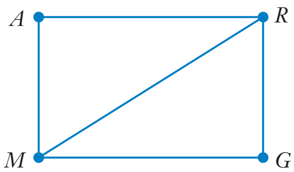
A A to C

B B to D

C B to C

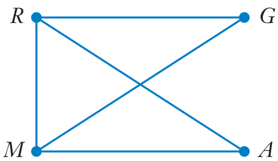
D A to B

3 This network diagram shows Facebook connections between Alex (A), Mariah (M), Georgia (G) and Rabibah (R). An edge represents a friendship between 2 people.

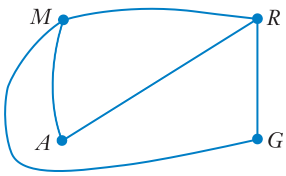


Which network diagram does *not* show the same information?

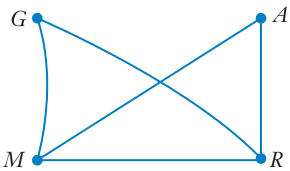
A



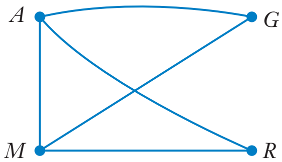
B



C



D



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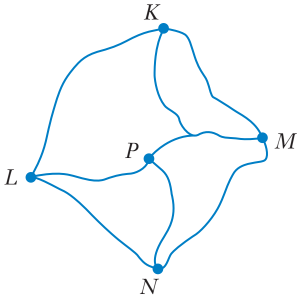
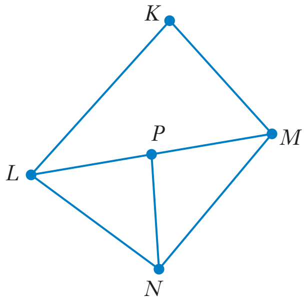
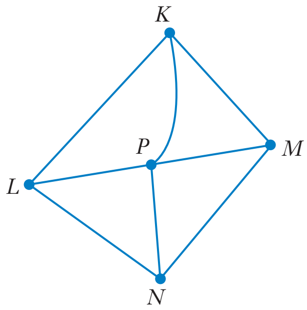
4 Which network diagram represents this map showing the roads linking 4 country towns?



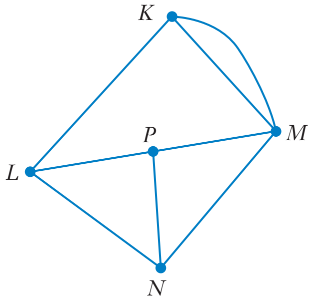
A



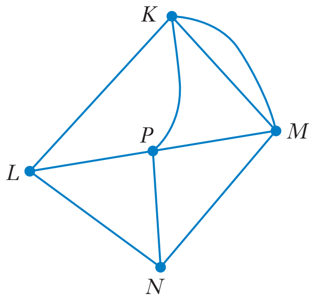
B



C



D



5 For each network, find:

i the weight of the network

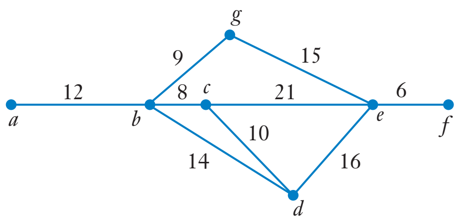
ii the shortest path from a to f

iii the longest path from a to f without using an edge more than once.

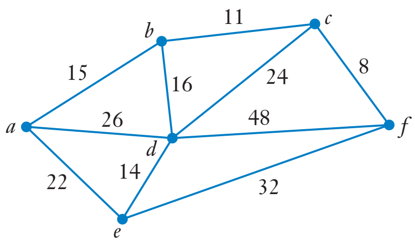
Example 1

WS Network diagrams

a



b



The worksheet 'Network diagrams' contains the diagrams from Questions 5, 6 and 13 for writing upon.

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Example 2

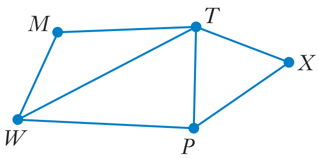
6 For each network:

i find the degree of each vertex

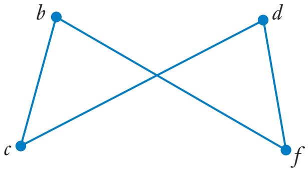
ii count how many vertices are even

iii show that the sum of the degrees is equal to twice the number of edges.

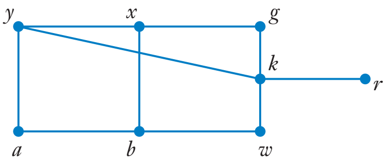
a



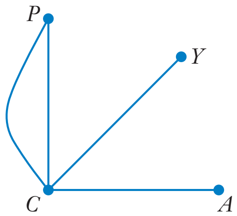
b



c



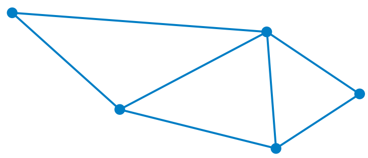
d



7 How many odd vertices has this network?

A 1 **B** 2

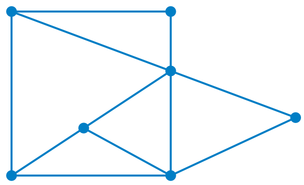
C 3 **D** 4



8 How many even vertices has this network?

A 2 **B** 3

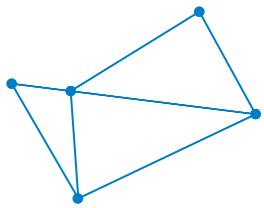
C 4 **D** 5



9 What is the sum of the degrees of all vertices in this network?

A 10 **B** 12

C 14 **D** 16



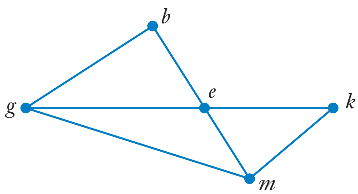
Example 3

10 **a** State whether each walk is a trail or path.

i *gbemke* **ii** *gmkeb*

iii *begmk* **iv** *ebgmke*

v *gbekmeg*



b Which of the walks in part **a** is:

i a circuit (a closed trail)? **ii** a cycle (a closed path)?

c Write down a closed walk that is not a cycle.

d Write down 2 cycles that use the edge *gm*.

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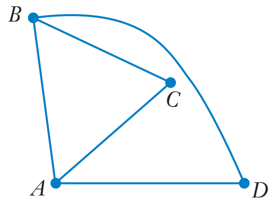
11 Which of the following walks is a cycle?

A *ABCD*

B *BCDAC*

C *CADBAC*

D *BCADB*



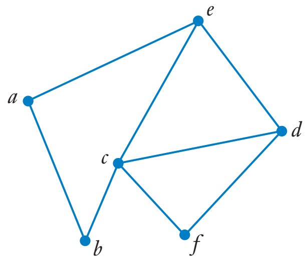
12 Which of the following walks is a trail?

A *abcedcf*

B *bcedcb*

C *abcdae*

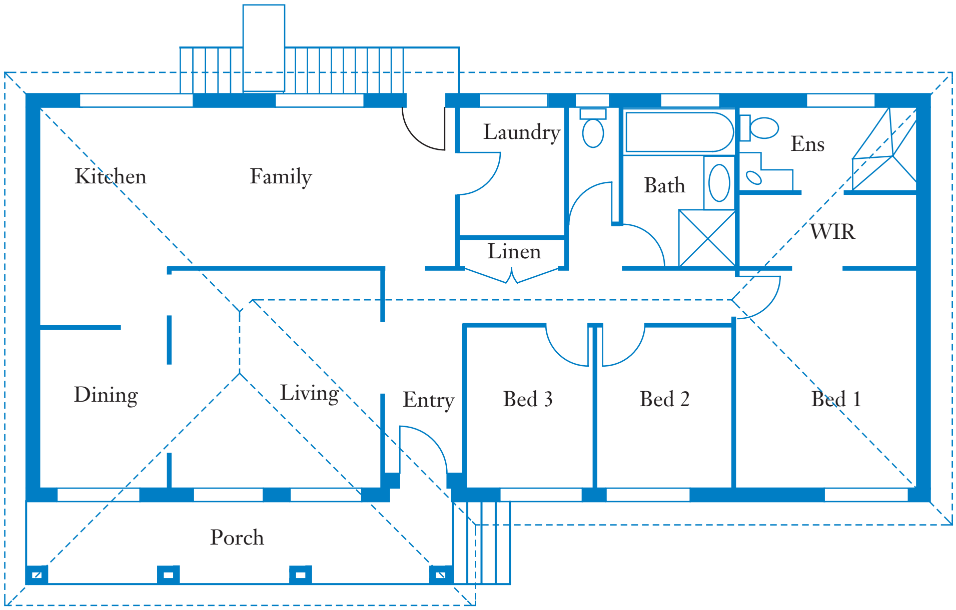
D *bcfdcba*



13 Model each house plan as a network, showing doors as edges and the rooms (including entry and hall) as vertices.

Example 4

a



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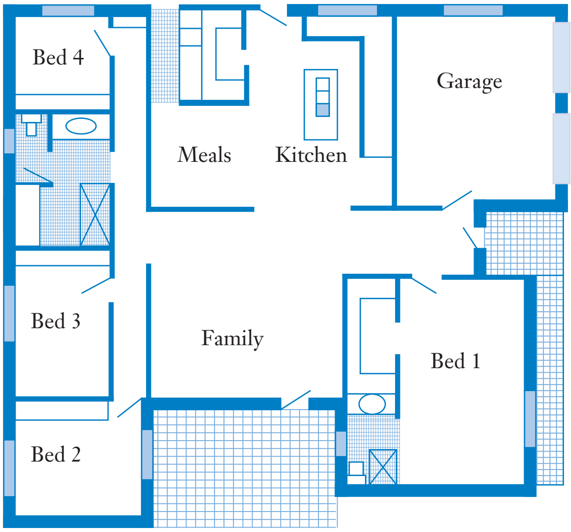
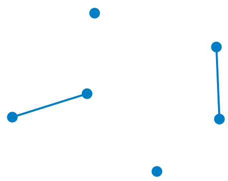
b



14 On the Sydney rail network, trains travelling from Campbelltown to Central go along 2 possible routes. They both stop at Glenfield, and then one branch passes through Liverpool, Lidcombe, Redfern and then Central, while the other branch passes through East Hills, Wolli Creek, International Airport and Domestic Airport, before reaching Central. From Central, all trains go around the City Circle, stopping at Town Hall, Wynyard, Circular Quay, St James, Museum and returning to Central.

Draw a network diagram that represents these routes.

15 What is the smallest number of edges required to make this diagram a connected network?



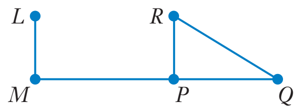
A 1

B 2

C 3

D 4

16 Five towns L, M, P, Q and R are linked by roads as shown by the network diagram, table and matrix.



	L	***M***	***P***	***Q***	***R***
L	0	1	0	0	0
M	1	0	1	0	0
P	0	*a*	0	1	1
Q	0	0	1	0	1
R	0	*b*	1	1	0

$$\begin{array}{c} \\ L \\ M \\ P \\ Q \\ R \end{array} \begin{array}{c} \begin{array}{ccccc} L & M & P & Q & R \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & a & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & b & 1 & 1 & 0 \end{bmatrix} \end{array}$$

a What is the meaning of a 0 in the table or matrix?

b Write the values of a and b.

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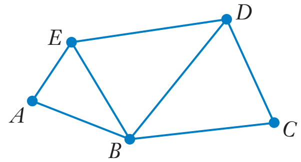
17 Copy and complete the table below for this network.

	A	B	C	D	E
A	0	1			
B			1		
C					
D					
E					



18 Draw the network that is represented by the table and matrix.

a

	A	B	C	D
A	0	1	1	1
B	1	0	1	1
C	1	1	0	1
D	1	1	1	0

b

$$\begin{array}{c} \\ A \\ B \\ C \\ D \\ E \end{array} \begin{array}{c} \begin{array}{ccccc} A & B & C & D & E \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

19 Draw the directed weighted network that is represented by the following.

Example 5

a

		To:				
		J	K	L	M	N
From:	**J**	–	–	–	–	–
	K	5	–	5	–	4
	L	–	4	–	8	4
	M	–	–	–	–	6
	N	–	–	6	–	–

b

To:

$$\text{From:}\quad \begin{array}{c} \\ A \\ B \\ C \\ D \end{array} \begin{array}{c} \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{bmatrix} - & 6 & 10 & 3 \\ 4 & - & 7 & 6 \\ - & 5 & - & - \\ - & - & 5 & - \end{bmatrix} \end{array}$$

20 The table shows the road distances (in km) between 9 towns in northern NSW.

	A	B	C	GI	Gr	T	U	W	Y
A	–	–	–	98	–	–	172	–	–
B	–	–	64	–	131	–	–	–	–
C	–	64	–	–	100	126	–	–	–
GI	98	–	–	–	164	93	–	129	–
Gr	–	131	100	164	–	–	110	–	–
T	–	–	126	93	–	–	–	–	193
U	172	–	–	–	110	–	–	–	–
W	–	–	–	129	–	–	–	–	85
Y	–	–	–	–	–	193	–	85	–

A – Armidale
B – Ballina
C – Casino
GI – Glen Innes
Gr – Grafton
T – Tenterfield
U – Uranga
W – Warialda
Y – Yetman

Draw a network diagram to represent the information in the table.

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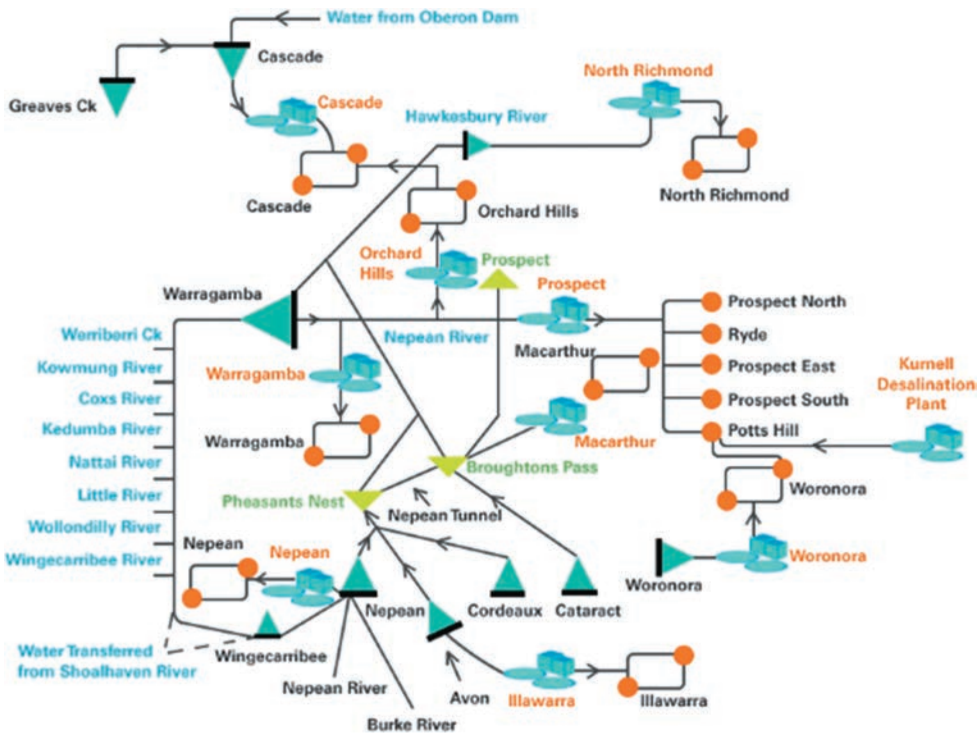
INVESTIGATION

USING NETWORKS

1 This network diagram shows the Sydney water supply.



Courtesy of Sydney Water

- Go to the **Sydney Water** website and access the water network.
- Obtain a copy of the water network diagram and show how water is supplied to:

  a Sydney **b** Blacktown.

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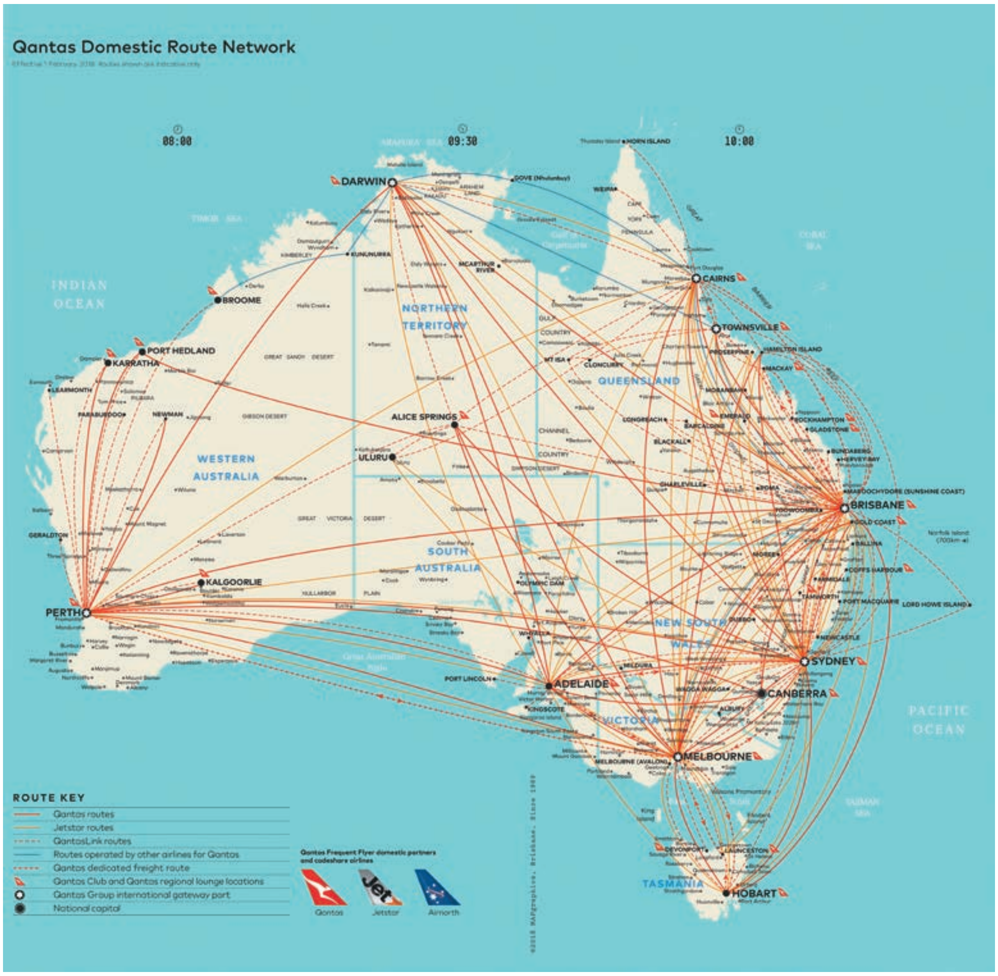
2 The network below shows the domestic air routes for Qantas.



Reproduced with permission from Qantas Airways Limited

a From which state and territory capital cities can you fly directly to every other capital city?

b From which capital cities can you fly directly to:

i Uluru? **ii** Lord Howe Island?

iii Broome? **iv** Cairns?

3 Network diagrams can also be used to represent social networks such as Facebook.

With a group of about 10 people in your class or Year 12, draw a diagram showing contacts through a social media platform such as Facebook and Instagram.

4 What is meant by the idea of '**six degrees of separation**'? Write a brief report on your findings and decide whether this could apply to you and any randomly selected student at your school.

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6.02 Eulerian trails and circuits

Eulerian trails

An **Eulerian trail** in a network is a walk that uses *every* edge in the network only once.

For an Eulerian trail to exist, the network must have exactly 2 odd vertices.

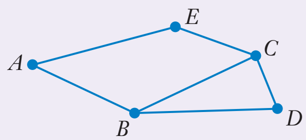
Eulerian trails must start at one odd vertex and finish at the other odd vertex.

Eulerian is pronounced 'Oiler-ian', and is named after the Swiss mathematician and engineer, Leonhard Euler (1707–1783).

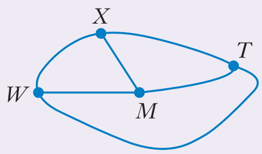
EXAMPLE 6

Does each network have an Eulerian trail?

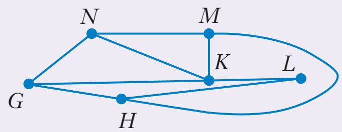
a



b



c



Solution

a The network has 2 odd vertices at B and C so it has an Eulerian trail, such as $CDBCEAB$.

b The network has all (4) odd vertices (degree 3) so it does not have an Eulerian trail.

c The network has 4 odd vertices (degree 3) so it does not have an Eulerian trail.

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Eulerian circuits

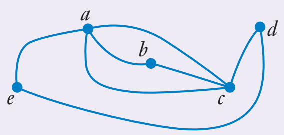
An **Eulerian circuit** in a network is a walk that uses *every* edge in the network only once and which **starts and ends** at the same vertex.

For an Eulerian circuit to exist, the vertices in the network must all be even.

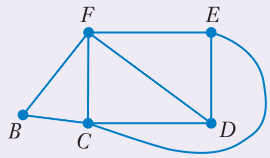
EXAMPLE 7

Does each network have an Eulerian circuit?

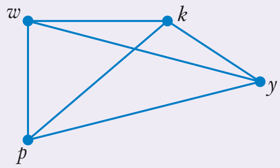
a



b



c



Solution

a The network contains an Eulerian circuit since all vertices are even (vertices b, d and e have degree 2, vertices a and c have degree 4). One possible Eulerian circuit is *abcdeaca*.

b Vertices D and E are odd (degree 3) so the network does not contain an Eulerian circuit.

c All vertices have degree 3, so the network does not contain an Eulerian circuit.

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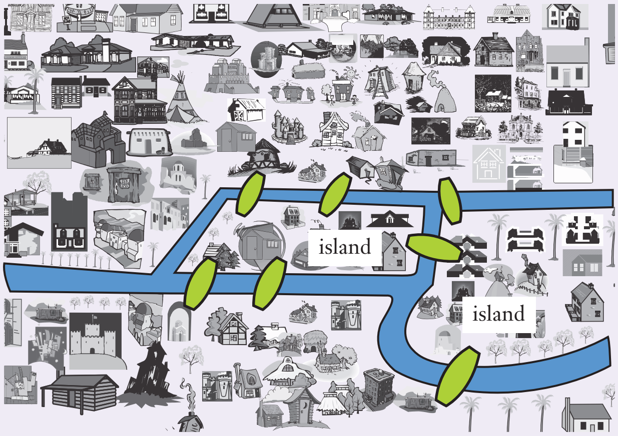
DID YOU KNOW?

The 7 Bridges of Königsberg

In the Prussian town of Königsberg (now Kaliningrad, Russia), there were 7 bridges connecting 2 islands and the banks of the Pregel River. There was a tradition where people would walk through the town on a Sunday and it became a challenge to complete a walk crossing each bridge only once. No person was able to do it but no one could prove that it was impossible.



In 1736, the Swiss mathematician Leonhard Euler solved the problem by reducing the map to a network. He found that it was impossible to do the walk. Eventually another bridge was built. That is why a walk that crosses every bridge exactly once is called an Eulerian trail.

1 **Draw a network to represent the map and explain why the original walk was impossible.**

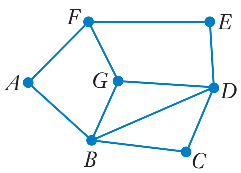
2 **An eighth bridge was built. Where would the bridge have been placed so that the network could be traversed?**

Exercise 6.02 Eulerian trails and circuits

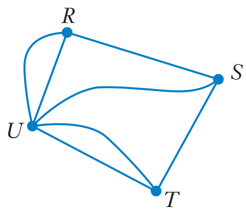
Example 6

1 Which network has an Eulerian trail? Select **A**, **B**, **C** or **D**.

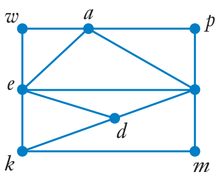
A



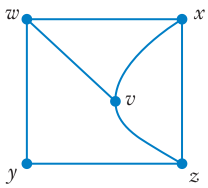
B



C



D



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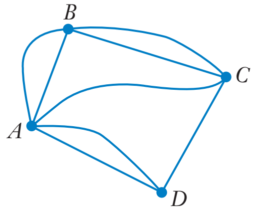
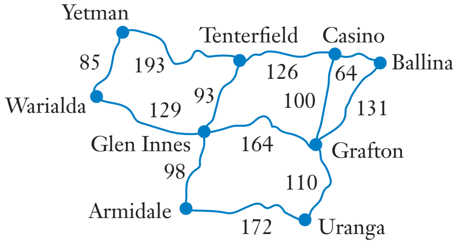
2 **a** Explain why this graph has an Eulerian trail.

b Copy the graph and show the trail.



3 The map shows the major roads between towns in northern NSW. Does the road network contain an Eulerian trail?

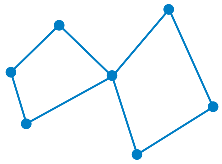
Give reasons.



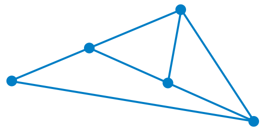
Example 7

4 Which network contains an Eulerian circuit? Select **A**, **B**, **C** or **D**.

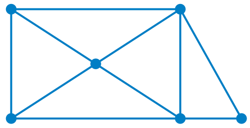
A



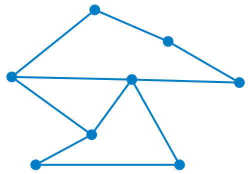
B



C



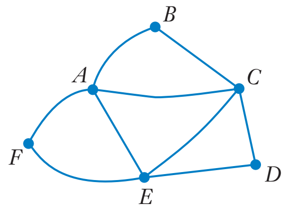
D



5 **a** Explain why the graph on the right has an Eulerian circuit.

b Write down a circuit.

c Copy the graph and show your circuit.

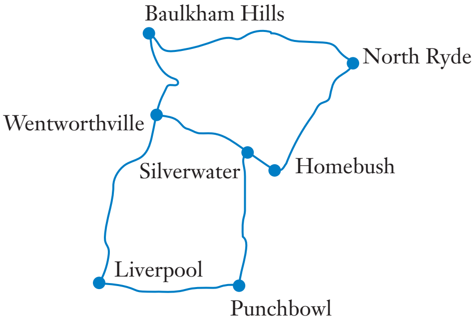


6 **a** Janet has to make deliveries to each place shown on the map. Can Janet make the deliveries using each road only once?

Give reasons.

b Is it possible for Janet to start and finish from the same place?

Give reasons.



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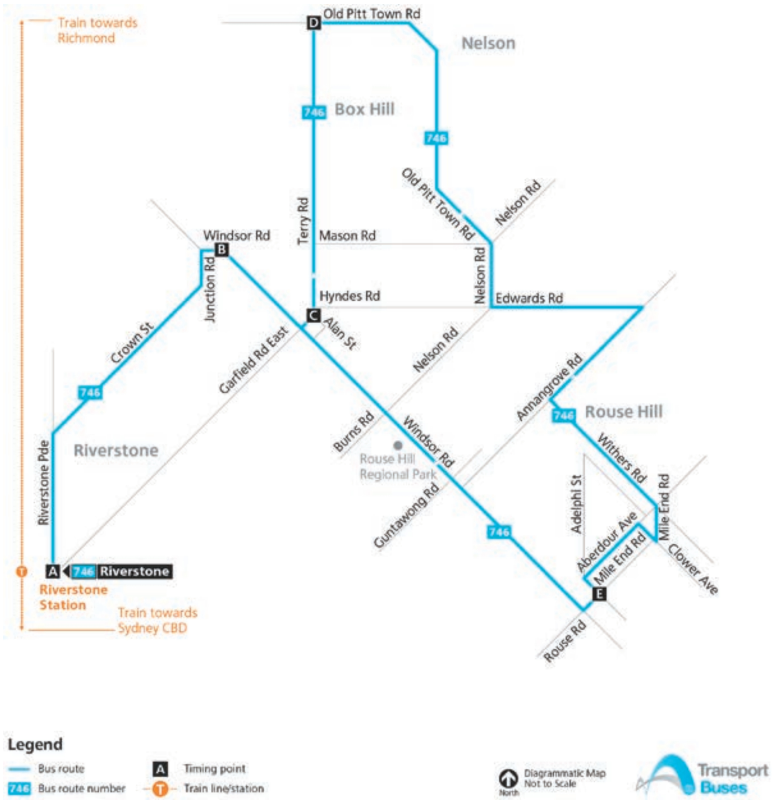
7 For each description of a network, decide whether it contains an Eulerian trail, an Eulerian circuit or neither.

- **a** A walk uses each edge of the network only once.
- **b** A walk starts and ends at the same vertex in the network.
- **c** There are no vertices of odd degree.
- **d** There are 2 vertices of odd degree.
- **e** There are more odd vertices than even vertices.
- **f** There is one even vertex and 2 odd vertices.
- **g** There are 3 even vertices and 2 odd vertices.
- **h** There are 5 odd vertices.
- **i** There are 6 even vertices and 6 odd vertices.
- **j** There are 5 even vertices and no odd vertices.

8 The blue diagram below is a route network map for a bus company in Blacktown.

- **a** Explain why the network map is an Eulerian trail but not an Eulerian circuit.
- **b** Draw a 'simplified' network diagram of the bus route.



Reproduced with permission from Busways Group Pty Ltd

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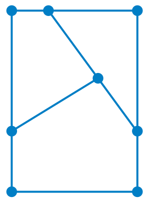
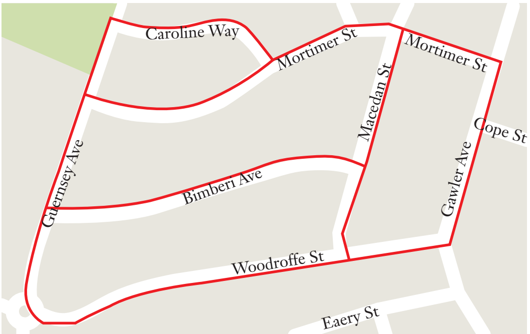
9 **a** The streets marked red on the map can be redrawn as a network diagram as shown.



Is it possible to deliver leaflets to every home by walking along each street only once? Give reasons.

b Redraw the roads shown on the map as a network and determine if Naheed can drive along each street only once.

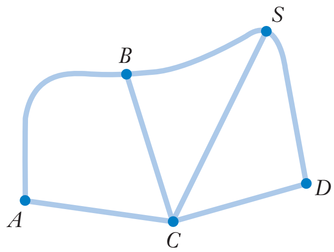


10 A line-marking truck is painting lines down the centre of roads shown in the network.

They start marking the lines from the intersection at S.

a Is it possible for the lines to be marked without going down a street twice?

b Explain why it is not possible for the truck to start and finish at S.



c An extra road between S and B is added to the network. Show that it is now possible to start and finish at S and write a route that could be used.

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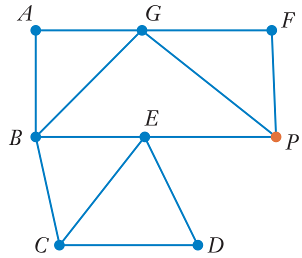
Eulerian trails and circuits

11 The street map for the delivery of local newspapers is drawn as a network. The route has to be organised so that the delivery person starts at P and does not drive along the same road twice.



a Explain why the delivery cannot start and finish at P.

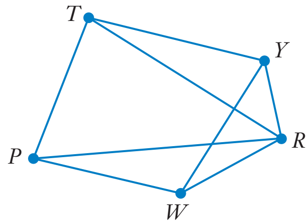
b A road is added to the network so that the delivery can start and finish at P. Which 2 vertices would the road join?

c Find a suitable route for the newspaper delivery.

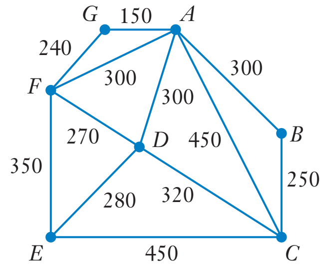
The worksheet 'Eulerian trails and circuits' contains the diagrams from Questions 11 to 15 for writing upon.

12 Crystal has to visit each city once on this network diagram, starting and ending at T.

Find possible routes Crystal could take.



13 This network represents a large park with walking paths. The vertices are rest areas and the numbers are the lengths of the paths in metres.



a Find the sum of the degrees of the vertices in the network diagram.

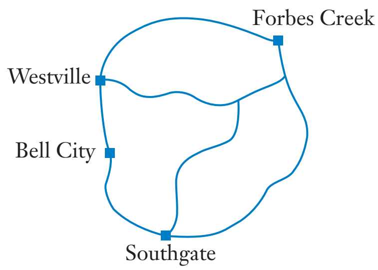
b Saieesha goes to the park and decides to walk along each path. Where should she start her walk?

c How far will Saieesha walk?

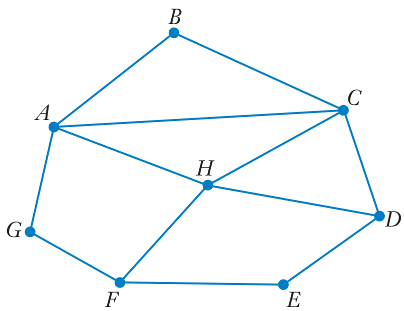
14 How many different ways can a person travel from Forbes Creek to Bell City without travelling along any road more than once? Select **A**, **B**, **C** or **D**.

A 5 **B** 6

C 7 **D** 8



15 Klaasville Tourist Centre publishes a map showing 8 'must-see' destinations for tourists, with the easiest routes shown to go from one destination to another.



a **i** Charissa and her family want to visit each destination, but use each road only once. They start from D. At which destination will they finish their trip?

ii How many destinations will Charissa and her family see exactly twice, no matter which route they take?

b The Roberts family decides they only wish to visit each destination once, starting at B. Write down the order in which they will see all the tourist destinations.

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16 If 2 people shake hands, a network diagram can be drawn where the vertices represent the people and an edge represents the handshake, as shown below.

Shutterstock.com/Hanna Kuprevich

So, for 2 people there is one handshake. Draw a network diagram to represent each number of people shaking hands with everybody else and count:

i the number of handshakes for each person

ii the total number of handshakes.

a 3 people shake hands

b 4 people shake hands

c 5 people shake hands

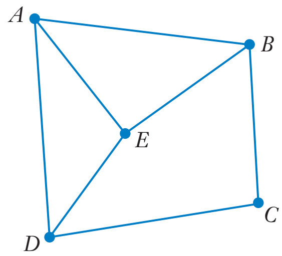
d 8 people shake hands

INVESTIGATION

HAMILTONIAN PATHS AND CIRCUITS

1 A **Hamiltonian path** is a path that visits *every* vertex only once.

For the network shown, a Hamiltonian path is ***ABEDC*** or ***ABCDE***.



a How many Hamiltonian paths can you find for this network?

b Are all the edges of the network included in a Hamiltonian path?

2 A **Hamiltonian circuit** is a Hamiltonian path that begins and ends at the same vertex.

For the network shown, a Hamiltonian circuit is ***KHJNMLK*** or ***HKLMNJK***.

How many Hamiltonian circuits can you find in this network?