

EPPING BOYS HIGH SCHOOL

Year 10 Stages 5.1–5.3 Mathematics — 2024 Term 1 Exam

FULL SOLUTIONS

SECTION A — MULTIPLE CHOICE (5 marks)

Question 1: Factorise $4 - (x + 1)^2$

Recognise this as a difference of two squares: $a^2 - b^2 = (a - b)(a + b)$

Here $a = 2$, $b = (x + 1)$:

$$4 - (x + 1)^2 = [2 - (x + 1)][2 + (x + 1)] = (1 - x)(3 + x)$$

Answer: D

Question 2: Area of triangle (sides 30 m and 20 m, included angle 35°)

Using the formula: $\text{Area} = \frac{1}{2} \times a \times b \times \sin C$

$$\text{Area} = \frac{1}{2} \times 30 \times 20 \times \sin 35^\circ = 300 \times 0.5736 = 172.07 \approx 172 \text{ m}^2$$

Answer: C

Question 3: Bearing of A from B

The plane flies on a bearing of 150° from A to B.

The reverse bearing (B back to A) = $150^\circ + 180^\circ = 330^\circ$.

Answer: D

Question 4: Garden dimensions

The outer rectangle has dimensions x (width) and y (height).

The path is 1 m on each side (left and right) and 1 m on the bottom. The top side is against the wall (no path).

Garden width = $x - 2$ (subtracting 1 m from each side).

Garden height = $y - 1$ (subtracting 1 m from the bottom only).

$$\text{Garden area: } (x - 2)(y - 1) = 50$$

Solving for y : $y - 1 = 50/(x - 2)$, so $y = 50/(x - 2) + 1$

Answer: C

Question 5: $\sqrt{32 + 10\sqrt{7}} = a + \sqrt{b}$

Square both sides: $(a + \sqrt{b})^2 = a^2 + 2a\sqrt{b} + b = 32 + 10\sqrt{7}$

Comparing rational parts: $a^2 + b = 32$

Comparing irrational parts: $2a\sqrt{b} = 10\sqrt{7} \rightarrow a\sqrt{b} = 5\sqrt{7}$

From the irrational part: $b = 7$, $a = 5$. Check: $25 + 7 = 32 \checkmark$

Answer: B (a = 5, b = 7)

SECTION B — RIGHT-ANGLED TRIANGLE TRIGONOMETRY (12 marks)

Question 1: Find x (2 marks)

Right-angled triangle: right angle at bottom-left, base = 10.6 cm, angle at bottom-right = $43^\circ 17'$, x = hypotenuse.

$$\cos(43^\circ 17') = \text{adjacent} / \text{hypotenuse} = 10.6 / x$$

$$x = 10.6 / \cos(43^\circ 17') = 10.6 / \cos(43.2833^\circ) = 10.6 / 0.72737$$

$$x \approx \mathbf{14.57 \text{ cm}}$$

Question 2: Find θ (2 marks)

Right-angled triangle: θ at top-left, right angle at top-right, horizontal side = 5, vertical side = 3.

$$\tan \theta = \text{opposite} / \text{adjacent} = 3 / 5 = 0.6$$

$$\theta = \tan^{-1}(0.6) = 30.9638^\circ$$

$$\text{Converting: } 0.9638 \times 60 = 57.8' \approx 58'$$

$$\theta \approx \mathbf{30^\circ 58'}$$

Question 3: Angle DE makes with plane ABCD (4 marks)

Step 1: Find EB.

In $\triangle ABE$: $\angle ABE = 90^\circ$, $\angle BAE = 30^\circ$, $AB = 60$ cm.

$$\tan 30^\circ = EB / AB \rightarrow EB = 60 \tan 30^\circ = 60 / \sqrt{3} = 20\sqrt{3} \text{ cm}$$

Step 2: Find DB (the projection of DE onto the plane).

ABCD is a square with side 60 cm. DB is the diagonal.

$$DB = \sqrt{(60^2 + 60^2)} = 60\sqrt{2} \text{ cm}$$

Step 3: E is directly above B (since $EB \perp$ plane ABCD).

The angle that DE makes with the plane = $\angle EDB$.

$$\tan(\angle EDB) = EB / DB = 20\sqrt{3} / (60\sqrt{2}) = \sqrt{3} / (3\sqrt{2}) = \sqrt{6} / 6 \approx 0.4082$$

$$\angle EDB = \tan^{-1}(0.4082) \approx \mathbf{22.2^\circ}$$

Question 4: Ship and cliff (2 + 2 marks)

(a) Diagram: A vertical cliff of 57 m. Observer at top. Two ships at sea-level.

First ship: angle of depression = 15° . Second ship (closer): angle of depression = 25° .

(b) Distance sailed:

Let the base of the cliff be O.

$$\text{First sighting: } \tan 15^\circ = 57 / d_1 \rightarrow d_1 = 57 / \tan 15^\circ = 57 / 0.2679 = 212.76 \text{ m}$$

$$\text{Second sighting: } \tan 25^\circ = 57 / d_2 \rightarrow d_2 = 57 / \tan 25^\circ = 57 / 0.4663 = 122.26 \text{ m}$$

$$\text{Distance sailed} = d_1 - d_2 = 212.76 - 122.26 = 90.50$$

$$\text{Distance} \approx \mathbf{91 \text{ m (to the nearest metre)}}$$

SECTION C — TRIGONOMETRY IN NON-RIGHT-ANGLED TRIANGLES (14 marks)

Question 1: $\sin \theta = 0.518$, θ acute (1 mark)

$$\theta = \sin^{-1}(0.518) \approx 31.2^\circ \text{ (or } 31^\circ 12')$$

$$\theta \approx 31^\circ 12'$$

Question 2: Sine rule to find x (2 marks)

Triangle with angles: 110° , 40° , and third angle = $180^\circ - 110^\circ - 40^\circ = 30^\circ$.

Side opposite 110° is 8. Side x is opposite 40° .

$$x / \sin 40^\circ = 8 / \sin 110^\circ$$

$$x = 8 \times \sin 40^\circ / \sin 110^\circ = 8 \times 0.6428 / 0.9397 = 5.4724$$

$$x \approx 5.5$$

Question 3: Find DC (2 marks)

A is at the top. ACB is a right angle at C. AB = 16 cm. $\angle ABC = 60^\circ$, $\angle ADC = 30^\circ$. D is left of C on the base.

Step 1: In right $\triangle ACB$: $\angle ACB = 90^\circ$, $\angle ABC = 60^\circ$.

$$AC = AB \sin 60^\circ = 16 \times (\sqrt{3}/2) = 8\sqrt{3} \text{ cm}$$

Step 2: In right $\triangle ACD$: $\angle ACD = 90^\circ$, $\angle ADC = 30^\circ$.

$$\tan 30^\circ = AC / DC \rightarrow DC = AC / \tan 30^\circ = 8\sqrt{3} / (1/\sqrt{3}) = 8\sqrt{3} \times \sqrt{3} = 24$$

$$DC = 24 \text{ cm}$$

Question 4: Flagpole problem (2 + 2 marks)

(a) Diagram: Flagpole of height 80 m at point O. A is due north, B is due west. $\angle AOB = 90^\circ$ (on level ground). Angle of elevation from A = 38° , from B = 47° .

(b) Calculate AB:

$$OA = 80 / \tan 38^\circ = 80 / 0.7813 = 102.39 \text{ m}$$

$$OB = 80 / \tan 47^\circ = 80 / 1.0724 = 74.60 \text{ m}$$

Since A is due north and B is due west, $\angle AOB = 90^\circ$.

$$AB^2 = OA^2 + OB^2 = 102.39^2 + 74.60^2 = 10483.7 + 5565.2 = 16048.9$$

$$AB = \sqrt{16048.9} \approx 127 \text{ m}$$

Question 5: Ship bearing problem (3 + 2 marks)

(a) From Sydney S, sail 200 km on bearing 040° to point P, then 345 km on bearing 157° to Q.

At P, the back-bearing to S = $040^\circ + 180^\circ = 220^\circ$.

Included angle at P = $220^\circ - 157^\circ = 63^\circ$.

By the cosine rule:

$$SQ^2 = 200^2 + 345^2 - 2(200)(345) \cos 63^\circ$$

$$= 40\,000 + 119\,025 - 138\,000 \times 0.4540$$

$$= 159\,025 - 62\,651 = 96\,374$$

$$\mathbf{SQ = \sqrt{96\,374} \approx 310 \text{ km}}$$

(b) Bearing of ship from Sydney:

Using the sine rule: $\sin(\angle S) / 345 = \sin 63^\circ / 310$

$$\sin(\angle S) = 345 \times \sin 63^\circ / 310 = 345 \times 0.8910 / 310 = 0.9910$$

$$\angle S = \sin^{-1}(0.9910) \approx 82^\circ$$

$$\mathbf{\text{Bearing of ship from Sydney} = 040^\circ + 82^\circ = 122^\circ}$$

SECTION D — ALGEBRAIC TECHNIQUES (13 marks)

Question 1(a): Factorise $a^2 - 5a - 14$ (1 mark)

Find two numbers that multiply to -14 and add to -5 : -7 and $+2$.

$$a^2 - 5a - 14 = (a - 7)(a + 2)$$

Question 1(b): Factorise $a^2 + 1/a^2 + 2$ (1 mark)

Recognise this as a perfect square:

$$a^2 + 2(a)(1/a) + (1/a)^2 = (a + 1/a)^2$$

$$= (a + 1/a)^2$$

Question 1(c): Factorise $5x^2 - 3x - 2$ (1 mark)

Product = $5 \times (-2) = -10$. Sum = -3 . Numbers: -5 and $+2$.

$$5x^2 - 5x + 2x - 2 = 5x(x - 1) + 2(x - 1) = (5x + 2)(x - 1)$$

$$= (5x + 2)(x - 1)$$

Question 1(d): Factorise $x^2 - y^2 - 4x + 4y$ (2 marks)

$$\text{Group: } (x^2 - y^2) - 4(x - y)$$

$$= (x - y)(x + y) - 4(x - y)$$

$$= (x - y)(x + y - 4)$$

Question 2(a): $(3x + 4)/5 + (x - 2)/2$ (2 marks)

LCD = 10:

$$= [2(3x + 4) + 5(x - 2)] / 10$$

$$= (6x + 8 + 5x - 10) / 10$$

$$= (11x - 2) / 10$$

Question 2(b): $a^2/(a - b) + b^2/(b - a)$ (3 marks)

Note that $(b - a) = -(a - b)$, so $b^2/(b - a) = -b^2/(a - b)$.

$$= a^2/(a - b) - b^2/(a - b)$$

$$= (a^2 - b^2) / (a - b)$$

$$= (a - b)(a + b) / (a - b)$$

$$= a + b$$

Question 2(c): Complex fraction multiplication and division (3 marks)

Expression: $[(x^2 - x)/(x^2 + x - 2)] \times [(x^2 - x - 2)/(x^2 + x)] \div [(x^2 + 4x + 4)/(x^2 - 4x + 4)]$

Factorise everything:

$$x^2 - x = x(x - 1) \quad | \quad x^2 + x - 2 = (x + 2)(x - 1)$$

$$x^2 - x - 2 = (x - 2)(x + 1) \quad | \quad x^2 + x = x(x + 1)$$

$$x^2 + 4x + 4 = (x + 2)^2 \quad | \quad x^2 - 4x + 4 = (x - 2)^2$$

$$= [x(x-1) / (x+2)(x-1)] \times [(x-2)(x+1) / x(x+1)] \times [(x-2)^2 / (x+2)^2]$$

Cancel common factors:

$$= [1/(x+2)] \times [(x-2)/1] \times [(x-2)^2/(x+2)^2]$$

$$= (x - 2)^3 / (x + 2)^3 \quad \text{or equivalently} \quad [(x-2)/(x+2)]^3$$

SECTION E — SURDS AND INDICES (13 marks)

Question 1(a): $\sqrt{12} + \sqrt{48} - \sqrt{108}$ (2 marks)

Simplify each surd:

$$\sqrt{12} = 2\sqrt{3} \quad | \quad \sqrt{48} = 4\sqrt{3} \quad | \quad \sqrt{108} = 6\sqrt{3}$$

$$= 2\sqrt{3} + 4\sqrt{3} - 6\sqrt{3} = 0$$

Question 1(b): $(\sqrt{a} - b\sqrt{b})^2$ (2 marks)

Expand using $(p - q)^2 = p^2 - 2pq + q^2$:

$$= (\sqrt{a})^2 - 2(\sqrt{a})(b\sqrt{b}) + (b\sqrt{b})^2$$

$$= a - 2b\sqrt{ab} + b^3$$

Question 2: Rationalise $\sqrt{8} / (3\sqrt{10})$ (2 marks)

$$= 2\sqrt{2} / (3\sqrt{10})$$

Multiply numerator and denominator by $\sqrt{10}$:

$$= 2\sqrt{2} \times \sqrt{10} / (3 \times 10) = 2\sqrt{20} / 30 = 2 \times 2\sqrt{5} / 30 = 4\sqrt{5} / 30$$

$$= 2\sqrt{5} / 15$$

Question 3: Prove $2/(2 + \sqrt{3}) + 12/(3 - \sqrt{3})$ is rational (3 marks)

First fraction: Rationalise $2/(2 + \sqrt{3})$:

$$= 2(2 - \sqrt{3}) / [(2 + \sqrt{3})(2 - \sqrt{3})] = 2(2 - \sqrt{3}) / (4 - 3) = 2(2 - \sqrt{3}) = 4 - 2\sqrt{3}$$

Second fraction: Rationalise $12/(3 - \sqrt{3})$:

$$= 12(3 + \sqrt{3}) / [(3 - \sqrt{3})(3 + \sqrt{3})] = 12(3 + \sqrt{3}) / (9 - 3) = 12(3 + \sqrt{3}) / 6 = 2(3 + \sqrt{3}) = 6 + 2\sqrt{3}$$

$$\text{Sum: } (4 - 2\sqrt{3}) + (6 + 2\sqrt{3}) = 10$$

Since 10 is a rational number, the expression is rational. ■

Question 4: Simplify $[(27x^4)^{-1/3} \times (3x^{1/3})^2] / [(3x)^0 \times 3(x^2)^{1/2}]$ (2 marks)

Numerator:

$$(27x^4)^{-1/3} = 27^{-1/3} \times x^{-4/3} = (1/3) \times x^{-4/3}$$

$$(3x^{1/3})^2 = 9x^{2/3}$$

$$\text{Numerator} = (1/3) \times x^{-4/3} \times 9x^{2/3} = 3x^{-2/3}$$

Denominator:

$$(3x)^0 = 1 \quad | \quad 3(x^2)^{1/2} = 3x$$

$$\text{Denominator} = 1 \times 3x = 3x$$

$$\text{Result} = 3x^{-2/3} / (3x) = x^{-2/3} / x = x^{-2/3 - 1} = x^{-5/3}$$

$$= x^{-5/3} \quad (\text{or equivalently } 1/x^{5/3})$$

Question 5: If $5^x = 2$, find 25^{2x+1} (2 marks)

$$25^{2x+1} = (5^2)^{2x+1} = 5^{2(2x+1)} = 5^{4x+2}$$

$$= 5^{4x} \times 5^2$$

$$= (5^x)^4 \times 25$$

$$= 2^4 \times 25 = 16 \times 25$$

$$= \mathbf{400}$$

— END OF SOLUTIONS —