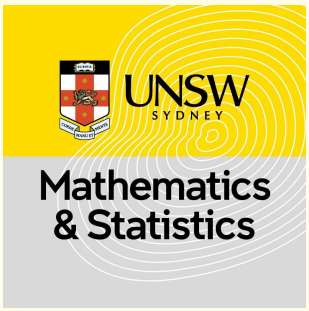


MATH5805
A Modern Introduction to Extreme Value Theory
Term 1, 2025

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Week 9 - Tutorial Solutions

Exercise 1

Write down the expression for the angular densities on each subspace of the simplex $\mathbb{W}_3$, that is, for dimension $d = 3$.

Solution:
The list of subspaces is provided in the Week 9 lecture, slide 36. We omit them here for brevity. To begin, we list the mass at each of the vertices. We have

$$H(\boldsymbol{e}_1) = -z_1^2 \lim_{z_2,z_3\to 0} \frac{\partial}{\partial z_1} V(\boldsymbol{z}),$$
$$H(\boldsymbol{e}_2), = -z_2^2 \lim_{z_1,z_3\to 0} \frac{\partial}{\partial z_2} V(\boldsymbol{z})$$
$$\text{and } H(\boldsymbol{e}_3) = -z_3^2 \lim_{z_1,z_2\to 0} \frac{\partial}{\partial z_3} V(\boldsymbol{z}).$$

For the edges, we have

$$h_{3,\{1,2\}}(\omega) = -\lim_{z_3\to 0} \frac{\partial^2}{\partial z_1 \partial z_2} V(\omega, 1-\omega, z_3),$$
$$h_{3,\{1,3\}}(\omega) = -\lim_{z_2\to 0} \frac{\partial^2}{\partial z_1 \partial z_3} V(\omega, z_2, 1-\omega),$$
$$\text{and } h_{3,\{2,3\}}(\omega) = -\lim_{z_1\to 0} \frac{\partial^2}{\partial z_2 \partial z_3} V(z_1, \omega, 1-\omega)$$

Finally, for the interior we have

$$h_{3,\{1,2,3\}}(\omega_1, \omega_2, 1-\omega_1-\omega_2) = -\frac{\partial^3}{\partial z_1 \partial z_2 \partial z_3} V(\omega_1, \omega_2, 1-\omega_1-\omega_2).$$

We've applied some notational abuse. You should read the solutions as taking the derivative first and then evaluating the resulting function at the relevant ω.

Exercise 2

Let $\bigwedge_{k=1}^d \omega_k^{-1} y_k = \min_{k=1,\dots,d} \omega_k^{-1} y_k$. Show that

$$\int_{\bigwedge_{k=1}^d \omega_k^{-1} y_k}^{\infty} \frac{1}{r^2}\,\mathrm{d}r = \max_{\ell=1,\dots,d} \frac{\omega_\ell}{y_\ell}.$$

Solution:
We first prove that $\max\{a_1, \ldots, a_n\} = \left(\min\{a_1^{-1}, \ldots, a_n^{-1}\}\right)^{-1}$ — *you may wish to show that this is true.* Now

$$\int_{\bigwedge_{k=1}^{d} \omega_k^{-1} y_k}^{\infty} r^{-2} \, \mathrm{d}r = \left[r^{-1}\right]_{\infty}^{\bigwedge_{k=1}^{d} \omega_k^{-1} y_k} = \left(\bigwedge_{k=1}^{d} \omega_k^{-1} y_k\right)^{-1} = \bigvee_{k=1}^{d} \frac{\omega_k}{y_k}.$$